An Efficient Iterative Solver Strategy for 2.5D Acoustic Simulation

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openCFS Annual Meeting 2026
Graz, April 9th, 2026

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Comparison of 3D and 2D Modeling

- 3D Model (The Real World)
  - Captures full geometry and physics
  - Complex sources (point/line/surface/volume)
  - **Challenge**: high computational cost
- 2D Model (The Simplification)
  - Requires both **geometry** and **loading** to be invariant in one direction
  - Very efficient, but limited applicability
  - **Largest limitation**: fails to model a 3D point source
  - Point source in 2D model = line source in 3D model

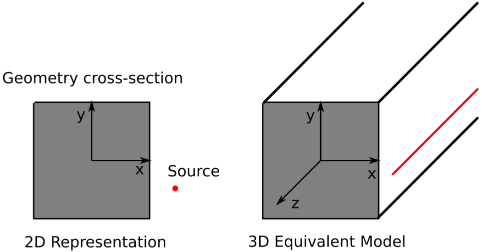


Figure: A 2D model with point source excitation and its 3D equivalent model.

2.5D Modeling (Wavenumber Domain Method)

- Fills the gap between 2D and 3D modeling
- Requires only **constant geometry** cross-section along one direction
- Based on spatial fourier transform, $p(x, y, z) \to \tilde{p}(x, y, k_z)$
- 3D problem $\to$ a series of **independently solvable** 2D problems

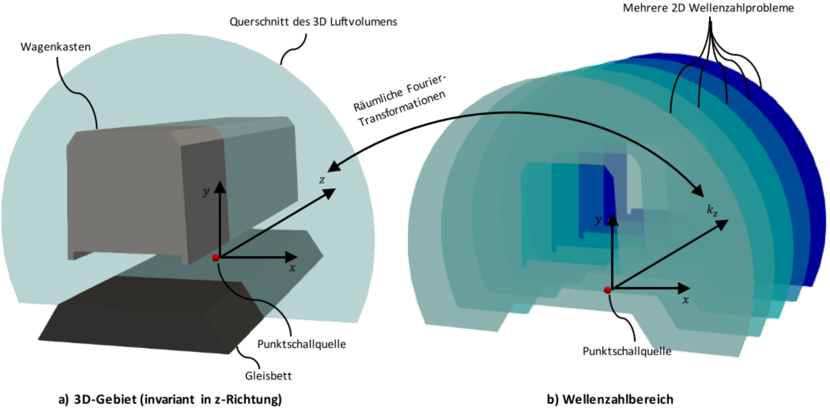
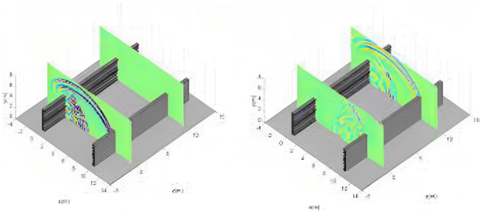


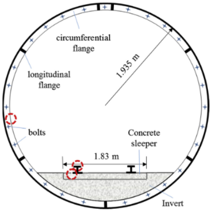
Figure: External sound wave propagation problem in railway acoustics.

Application Examples of 2.5D Method

- Railway Acoustics
  - LI et al. 2020: Using a 2.5D boundary element model to predict the sound distribution on train external surfaces due to rolling noise
  - LI et al. 2021: A 2.5D acoustic finite element method applied to railway acoustics
- Noise Barrier
  - DUHAMMEL 1996: EFFICIENT CALCULATION OF THE THREE-DIMENSIONAL SOUND PRESSURE FIELD AROUND A NOISE BARRIER
  - GAJARDO et al. 2015: Numerical Analysis of Acoustic Barriers with a Diffusive Surface Using a 2.5D Boundary Element Model
- Structural Vibration
  - JEAN, VILLOT 2015: A comparison of 2D, 2.5D and 3D BEM models for the study of railway induced vibrations
  - JIN et al. 2018: A 2.5D finite element and boundary element model for the ground vibration from trains in tunnels and validation using measurement data



(a) GAJARDO et al. 2015



(b) JIN et al. 2018

2.5D Finite Element Formulation[1]

Helmholtz-Gleichung für lineare Akustik im Frequenzbereich

3D Gradient, Schalldruck

$$\nabla^2 p + k^2 p = 0 \quad (1)$$

k: Die der Frequenz entsprechende Wellenzahl

Räumliche Fourier-Transformationen

Fourier-Transformierte in z-Richtung; Wellenzahl in z-Richtung

$$\tilde{p}(x,y,k_z) = \int_{-\infty}^{+\infty} p(x,y,z) e^{ik_z z}\, dz \quad (2)$$

2D Ortskoordinaten; Imaginäre Einheit

Physikalischer Schalldruck im Raum

$$p(x,y,z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{p}(x,y,k_z) e^{-ik_z z}\, dk_z \quad (3)$$

3D Ortskoordinaten

① Ansatz einsetzen

② Einführung der Testfunktion p' und über das gesamte Rechengebiet integrieren

Schwache Formulierung der 2.5D Akustik FEM-Gleichung

2D Gradient; Testfunktion; Normalenvektor

$$\int_A [-\nabla_\perp p' \cdot \nabla_\perp \tilde{p} + (k^2 - k_z^2) p' \tilde{p}]\, \mathrm{d}x\, \mathrm{d}y + \int_\Gamma p' \nabla_\perp \tilde{p} \cdot \boldsymbol{n}\, \mathrm{d}\Gamma = 0 \quad (4)$$

2D Flächenintegral; 1D Randintegral

③ Galerkin-Verfahren anwenden

Diskretes lineares System

2D Massenmatrix; 2D Dämpfungsmatrix

$$[(k_z^2 c_0^2 - \omega^2)\boldsymbol{M} + i\omega \boldsymbol{C} + \boldsymbol{K}]\tilde{\boldsymbol{p}} = \boldsymbol{F} \quad (5)$$

Schallgeschwindigkeit; 2D Steifigkeitsmatrix; Akustische Erregung

Die der Frequenz entsprechende Winkelgeschwindigkeit

④ Inverse Fouriertransformation anwenden

[1] Li et al: A 2.5D acoustic finite element method applied to railway acoustics. Applied Acoustics, Vol. 182, 2021.

Wavenumber Spectrum of 2.5D Simulation

$$\left[\underbrace{(k_z^2 c_0^2}_{\omega_{k_z}^2} - \omega_{3D}^2)\mathbf{M} + i\omega_{3D}\mathbf{C} + \mathbf{K}\right] \tilde{\boldsymbol{p}} = \boldsymbol{f}$$

$$\tilde{\mathbf{A}}\left(\omega_{k_z}\right) \tilde{\boldsymbol{p}}\left(\omega_{k_z}\right) = \boldsymbol{f}$$

- Wavenumber spectrum: frequency sweep of axial wavenumber k_z for a fixed excitation frequency ω_{3D}
- Tunnel wall absorptions modeled as surface impedance BC $\rightarrow$ system damping matrix $\mathbf{C}$

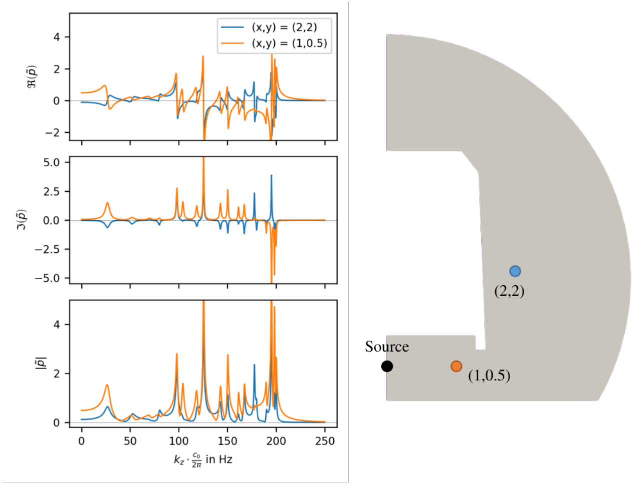
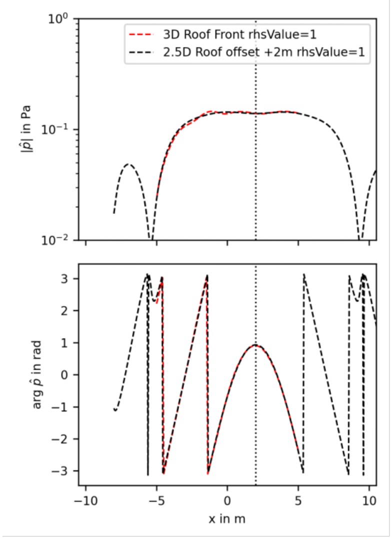


Figure: Left: Wavenumber spectrum at representative positions. Right: Train inside a tunnel with absorption on the walls.

Physical Sound Pressure in 3D Space



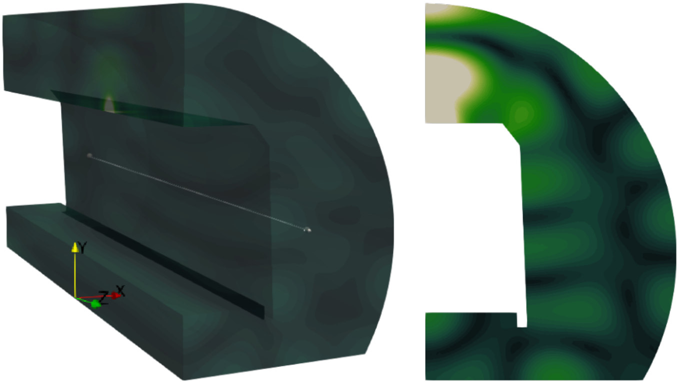


Figure: 2.5D results vs 3D reference model, point source excitation on the roof of car body.

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Motivation

- $[(k_z^2 c_0^2 - \omega_{3D}^2)\mathbf{M} + i\omega_{3D}\mathbf{C} + \mathbf{K}]\tilde{\boldsymbol{p}} = \boldsymbol{f}$, $\tilde{\mathbf{A}}(\omega_{k_z})\,\tilde{\boldsymbol{p}}(\omega_{k_z}) = \boldsymbol{f}$
- Solved using direct solver (e.g. PARDISO, MUMPS) for each wavenumber ω_{k_z}
- **Drawback**: High computational cost for matrix factorization
- **Suggested Improvement**: Reuse $\mathbf{LDL}^\mathrm{T}$ of $\tilde{\mathbf{A}}(\omega_{k_z})$ as preconditioners for iterative solving of $\tilde{\mathbf{A}}(\omega_{k_z} \pm \omega_{k_z,\mathrm{offset}})$

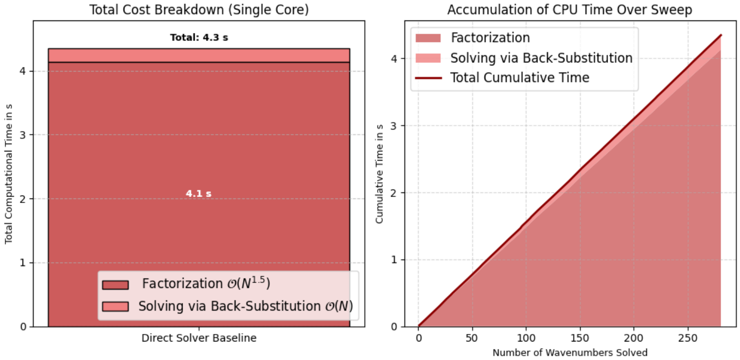

Figure: Computational cost for direct solving strategy using MUMPS ($\mathbf{LDL}^\mathrm{T}$-Factorization)

Choice of Iterative Solvers

$$\left[\underbrace{\left(k_z^2 c_0^2\right.}_{\omega_{k_z}^2} - \omega_{3D}^2 \Big) \mathbf{M} + i\omega_{3D}\mathbf{C} + \mathbf{K} \right] \tilde{\boldsymbol{p}} = \boldsymbol{f}$$

$$\tilde{\mathbf{A}}\left(\omega_{k_z}\right) \tilde{\boldsymbol{p}}\left(\omega_{k_z}\right) = \boldsymbol{f}$$

Properties of dynamic stiffness matrix $\tilde{\mathbf{A}}$:

- Sparse, square and symmetric (FEM, linear acoustics)
- Complex-valued and Non-Hermite (due to prefactor of **C**)
- Indefinite (depending on ω_{k_z})

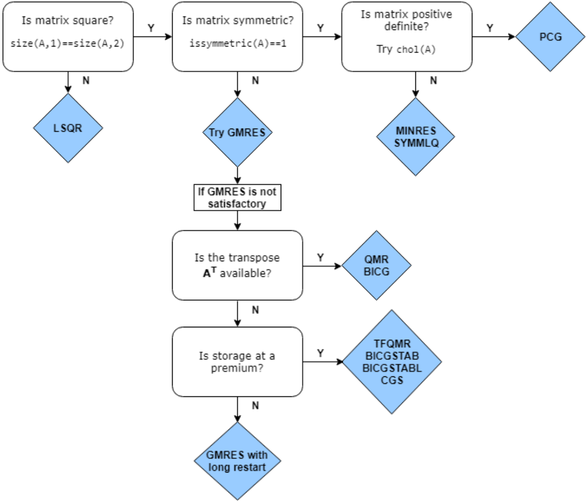

Figure: Flow chart for choosing an iterative solver for linear system[1]

[2] https://de.mathworks.com/help/matlab/math/iterative-methods-for-linear-systems.html

Efficient Iterative Solver Strategy

For Fast Frequency Sweep of 2.5D Wavenumber Spectrum for Single ω_{3D}

1. Assemble the 2D FE matrices **M**, **C**, **K**, **F** using a FE-solver, e.g. the Almighty openCFS orz
2. Choose a initial wavenumber $\omega_{k_z,\text{init}}$, e.g. the geometrical center of the wavenumber spectrum $\left[0, \omega_{k_z,\text{cutoff}}\right]$ i.e. $\omega_{k_z,\text{init}} = \frac{1}{\sqrt{2}} \cdot \omega_{k_z,\text{cutoff}}$
3. Built the corresponding dynamic stiffness matrix $\tilde{\mathbf{A}}\left(\omega_{k_z,\text{init}}\right)$
4. Compute the $\mathbf{LDL}^{\text{T}}$ of $\tilde{\mathbf{A}}\left(\omega_{k_z,\text{init}}\right)$ with a direct solver e.g PARDISO or MUMPS
5. Pass **L** and **D** matrices as preconditioner to a suitable iterative solver, e.g. Bi-CGSTAB (general Matrix), COCG (symmetric Matrix)
6. Frequency sweep for $\left[0, \omega_{k_z,\text{init}}\right)$ and $\left(\omega_{k_z,\text{init}}, \omega_{k_z,\text{cutoff}}\right]$ using the chosen preconditioned iterative method

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Computation Time

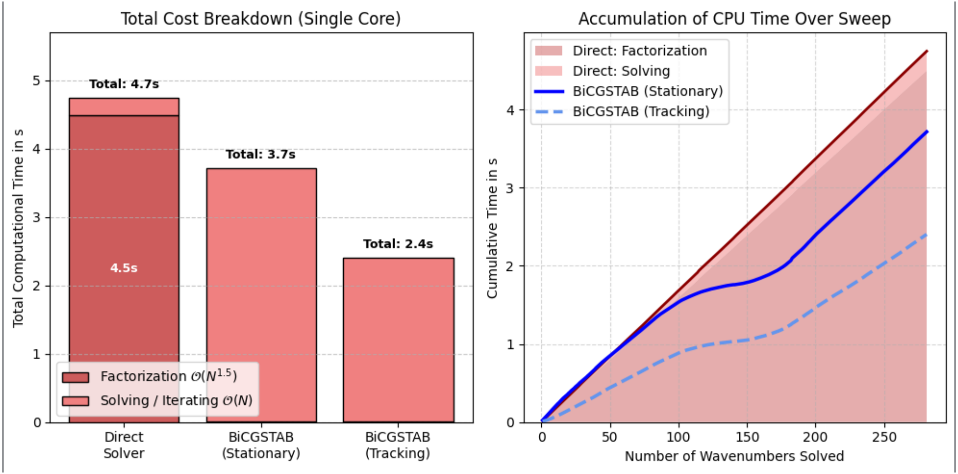

Figure: Comparison of computation time between direct solver and proposed iterative solving strategy. Stationary: static initial guess. Tracking: dynamically updated initial guess. Stopping Criteria for iterative solver is $\epsilon_{\text{tol,relative}} = 1e^{-3}$.

Residual Error

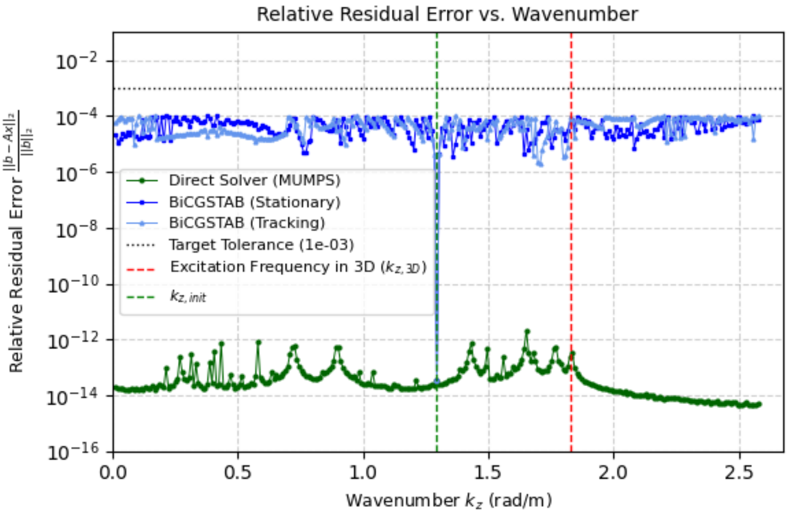

Figure: Comparison of residual error between direct solver and proposed iterative solving strategy.

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Summary and Discussion

Summary:

- Suggested preconditioned iterative solver strategy seems to work
- Further study to improve efficiency, e.g. convergence rate, storage, etc.
- Implementation in openCFS planned

Discussion Part I:

- Any questions regarding the contents so far?

Planned Implementation in openCFS

- Find an iterative solver library that
  1. supports user-defined preconditioner
  2. has at least Bi-CGSTAB, ideally COCG (due to symmetry of $\tilde{\mathbf{A}}$)
- Current candidate: the Ginkgo library (BiCGSTAB)
- Compute $\mathbf{LDL}^T$ decomposition with Intel PARDISO
- **New implementation**: Write an interface to pass **L** and **D** matrices to e.g. Ginkgo

Discussion Part II:

- Any useful tips, kindly reminders for the planned implementations?